A broadband electromagnetic homogenization method for composite materials

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This paper focuses on broadband electromagnetic (EM) modeling of composite materials. A homogenization method is carried out using 3D Finite Element calculations of transmission and reflection coefficients over a large frequency band (up to 60 GHz). Inversion techniques are then applied in order to extract electromagnetic properties. This approach extends the application domain of classical homogenization methods to include more types of inclusions thus making it possible to model woven composites.

***Index Terms*— 3D Finite Element Method (FEM), Composite materials, Electromagnetic properties, Homogenization.**

I. Introduction

THE automotive industry has been increasingly replacing metal alloys with composite materials due to their lighter weight and robust mechanical properties. However, using these materials in EM shielding applications requires modeling different types of inclusions to study their behavior and calculate the effective electrical properties. Because composites are heterogeneous materials, there exists a contrast in EM properties between the matrix (that holds the material together) and the inclusion particles (fibers, ellipsoids, spheres, etc.) that constitute them, which gives each phase a different EM response and makes the composite's general behavior complicated to model. This explains the need for homogenization methods that can describe their global electromagnetic response, by defining a homogeneous medium that shows an identical reaction when illuminated by an EM wave of a certain frequency.

In general, analytical homogenization methods exist, but their use is conditioned by multiple factors (shape of the inclusion, volume fraction, frequency, etc.). In [1], a comparative study is conducted on the calculation of effective permittivity using analytical methods, explaining their advantages and limits. While these methods have been proven effective for composites with spherical inclusions or unidirectional fibers, they do not apply in the case of woven composites. For this type of reinforcements the work in literature has relied on the periodicity that they present in order to use numerical method and therefore extract the material's shielding efficiency as done in [2] using a rigorous coupled wave analysis technique. Moreover, in [3] and [4], multiple experimental methods are used to measure the S parameters of a composite plate which can lead to explain the global behavior of the material.

In this paper, we combine a finite element (FEM) approach with an inversion technique to homogenize composite plates. First, the theory behind this work is presented: the model used for the simulations is described as well as the adopted inversion method. Then, the implementation results are analyzed and a conclusion is drawn.

II. Homogenization Method

A. 3D FEM Model

The first step to calculating the effective EM properties is to get the transmission and reflection coefficients in the frequency band of interest.

One of the main goals is to consider different shapes of inclusions, so a finite element software was adopted as a simulation tool. The 2D model described in [5] to calculate the shielding effectiveness of composites is expanded into a 3D model. Fig. 1 shows the model with cylindrical inclusions that can be replaced with any other geometry, thus making this model an adaptable one.

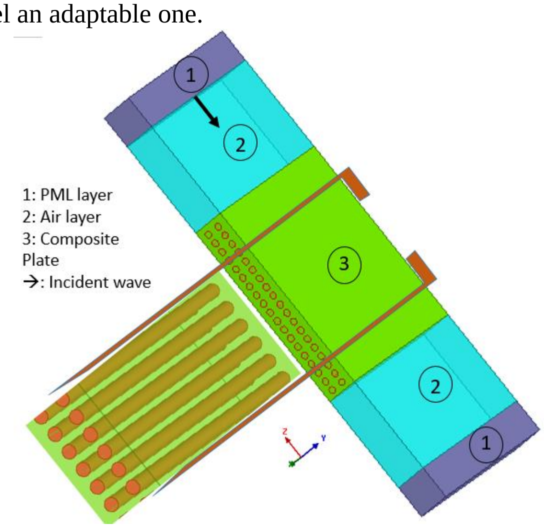

Fig. 1. 3D FEM model: Composite plate with cylindrical inclusions

The composite plate is surrounded by air allowing the EM wave to propagate. Neumann boundary conditions are applied to the appropriate surfaces to model long fibers, and Perfectly Matched Layers (PMLs) are added to simulate infinite domains.

B. Retrieving EM properties from S-parameters

The mostly used inversion method is the one developed by Nicolson, Ross and Weir in [6]. It relies on retrieving the complex permittivity and permeability of a plate using empirical formulas which relate S parameters with material properties. However, knowing in advance some properties about the material in question can help simplify the problem. For example, a non-magnetic material having a unity magnetic permeability lowers the number of unknowns to two. In this case, many optimization algorithms can be used to minimize a cost function F_c as in (1) thus finding the optimal electric permittivity and conductivity.

$$F_c = \left| S_{21SIM} - S_{21CAL}(\varepsilon_r^*) \right| \quad (1)$$

The search for the value of ε_r^* that will bring to a minimum the difference between the simulated transmission coefficient (S_{21SIM}) and the theoretical one (S_{21CAL}) is done using a genetic optimization algorithm.

III. Modeling Results

The FEM model detailed in II-A was applied to a 6mm thick composite plate with 30 rows of cylindrical inclusions having a 25µm radius each and occupying a volume fraction of 19.63%. Table I specifies the EM properties of each phase.

TABLE I : EM Properties Of The Composite Plate

Phase	Relative Permittivity ε_r	Conductivity σ [S/m]	Relative Permeability μ_r
Matrix	1	0	1
Inclusions	5	1000/4000/100000	1

The incident wave is propagating along the z-axis perpendicularly to the fibers, according to (2) where k_0 is the propagation constant and u_x is the unit vector of the x-axis.

$$E_{inc} = E.e^{j(\omega t - k_0 z)} u_x \quad (2)$$

The transmission coefficients $|S_{21}|$ [dB] are shown in Fig. 2.

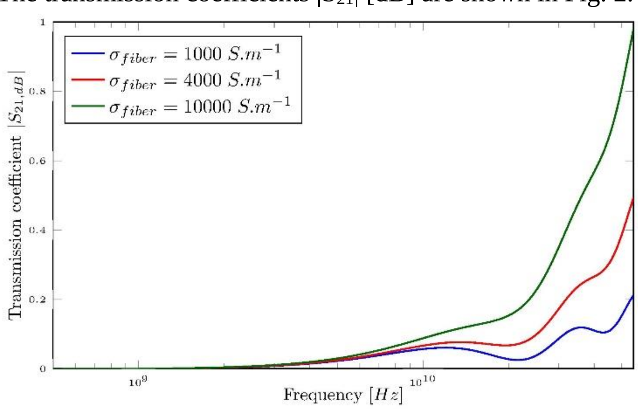

Fig. 2: Transmission coefficient (in dB) of the composite plate for different fiber conductivities as a function of the frequency (in Hz)

The global behavior of the plate at hand can be assimilated to that of a homogeneous one having the same effective properties. For such plate, the $|S_{21,dB}|$ coefficient (shielding efficiency) can be calculated using formulas in (3)-(4), where it is shown that this coefficient is influenced especially by the plate's thickness l and the material's properties also embedded in the index n.

$$S_{21} = e^{jkl}(1 - qe^{-2jkl}) / p \quad (3)$$

$$q = \left(\frac{n/\mu_r - 1}{n/\mu_r + 1}\right)^2 \text{ and } p = \frac{4n/\mu_r}{(n/\mu_r + 1)^2} \quad (4)$$

For this next step, the permeability and effective permittivity are assumed to be known. An optimization algorithm is applied to (1) and the effective conductivities (σ) are calculated at each frequency in order to minimize this function. Fig. 3 shows the results which demonstrate a behavior similar to that of the shielding efficiency in fig. 2.

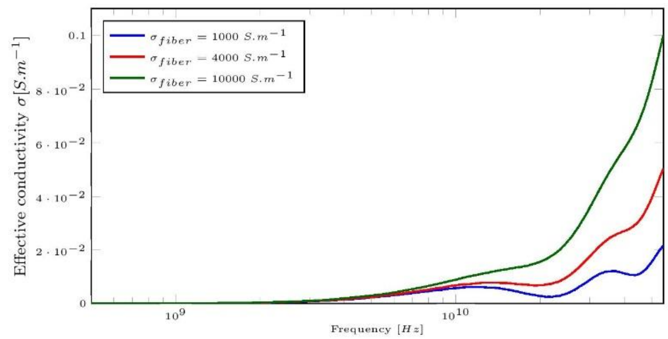

Fig. 3: Effective conductivity of the plate vs. frequency

IV. Conclusion

In this paper, a broadband homogenization method is carried out on a composite plate with cylindrical inclusions and the effective conductivity is extracted. This is done by simulating the transmission coefficient and minimizing the difference between this simulated coefficient and that of a homogeneous plate obtained using an empirical formula.

This work paves the path to modeling and homogenizing woven composites which will be presented in the full paper, thus breaking the limits of classical mixing rules that can only model composites with low volume fraction at relatively low frequencies.

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